GEOPHYSICAL RESEARCH LETTERS, VOL 21, NO. 12, PAGES 1177-1180, JUNE 15, 1994

A theoretical model of cooling viscous gravity currents with temperature-dependent viscosity

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Abstract. Gravity current theory has applications to any geophysical phenomena involving the spreading of fluid on a horizontal interface. Many geological gravity currents (e.g., lava flows and mantle plume heads) are composed of cooling fluid with temperature-dependent viscosity. An axisymmetric gravity current theory accounting for these thermo-viscous effects is thus presented and explored here. Unlike isoviscous gravity currents [*Huppert*, 1982], cooling variable-viscosity currents do not conserve shape and can undergo a somewhat exotic evolution. For large viscosity contrasts between cold and hot fluid, a constant volume, initially hot, domed current collapses rapidly into a flat plateau with a steep edge. Gravity currents ejected at a constant volume flux from a central conduit also spread with a flattened plateau shape. Continuously fed currents that have a large hot initial volume develop an outwardly propagating interior plateau. Regardless of initial state, continuously-fed, variable-viscosity currents grow primarily by thickening; this contrasts signficantly from isoviscous currents which grow almost entirely by spreading.

Introduction

Viscous gravity currents – i.e., fluid masses spreading under their own weight – are widely studied phenomena with applications to a large number of geological problems, from lava flows on Earth [e.g., *Crisp and Baloga*, 1990; *Fink and Griffiths*, 1992] and Venus [*McKenzie et al.*, 1992], to buoyant diapirs or plume-heads spreading beneath a rigid surface [*Olson*, 1990]. Symmetrically spreading isoviscous gravity currents are well described by analytical similarity theory [*Huppert*, 1982]; i.e., they conserve their basic shape as they spread. For many geological gravity currents, however, the fluid is often cooling from an initially hot state, or being fed by a source of hot fluid. Examples of this are lava flows and mantle plume heads. Moreover, many geological fluids have temperature-dependent viscosities wherein the fluid becomes weaker as its temperature increases. The combined effects of cooling and temperature-dependent viscosity lead to significant deviations from the basic lens-shaped similarity profiles of the constant viscosity case. In particular, fluid near the perimeter of the flow is cooler and more viscous than at the flow center, and thus can act as a plug causing the interior of the flow to inflate and/or change shape radically. In this paper we examine a basic nonlinear axisymmetric gravity current theory which accounts for these thermo-viscous effects.

Paper number 94GL01124
0094-8534/94/94GL-01124$03.00

Theory

The theoretical model describes a cylindrically axisymmetric, radially flowing, hot fluid mass cooling through two horizontal boundaries, at least one of which is deformable. The fluid viscosity is

$$\eta(\theta) = \frac{\eta_h \eta_c}{\eta_h + (\eta_c - \eta_h)\theta} \tag{1}$$

where η_h and η_c are the viscosities of the hottest and coldest fluid, respectively (thus $\eta_h \leq \eta_c$) and θ is the dimensionless temperature. Although this rheology is simplified, the inverse dependence of viscosity on temperature captures the essential behavior of viscous fluids; in particular, viscosity variations are largest where the mean temperature is coldest. For simplicity, we impose the same isothermal condition of $\theta = 0$ at both upper and lower boundaries (heights $z = 0$ and $z = H$, where H is the current's thickness); elsewhere in the fluid $0 < \theta \leq 1$. Thus, to first order

$$\theta = 6\Theta \frac{z}{H}\left(1 - \frac{z}{H}\right) \tag{2}$$

where $\Theta = \frac{1}{H}\int_0^H \theta dz$. Higher order contributions to the temperature may occur, especially near the center of the current, but (2) must always represent the dominant contribution. Radial motion of the current is described by the axisymmetric Stokes equation for shallow-layer flow with the hydrostatic and long-wavelength approximations (and thus vertical variations in viscous stress provide the dominant resistive force):

$$0 = -\Delta\rho g \frac{\partial H}{\partial r} + \frac{\partial}{\partial z}\left(\eta \frac{\partial v_r}{\partial z}\right) \tag{3}$$

where g is gravity, v_r is radial velocity and $\Delta\rho$ is the absolute density contrast across the deforming boundary. Because $\Delta\rho$ is treated as a constant, the gravity current is assumed to have an intrinsic density contrast with its surroundings which is much larger than thermal density variations. The vertically averaged v_r is

$$U = \frac{1}{H}\int_0^H v_r dz = -\frac{\Delta\rho g H^2}{C\eta_h\eta_c}(\eta_h + \eta'\Theta)\frac{\partial H}{\partial r} \tag{4}$$

The dimensionless constant C is either 3 if one horizontal boundary is free-slip and the other no-slip (as appropriate for surface gravity currents, e.g., lava flows), or 12 if both boundaries are no-slip (appropriate for low viscosity diapirs in a high-viscosity matrix). Similarly, $\eta'/(\eta_c - \eta_h)$ is $\frac{9}{10}$ if one boundary is free-slip the other no-slip, or $\frac{3}{5}$ if both boundaries are no-slip. Conservation of mass requires

$$\frac{\partial H}{\partial t} + \frac{1}{r}\frac{\partial (rUH)}{\partial r} = 0 \qquad (5)$$

and temperature transport is described by

$$\frac{\partial \Theta}{\partial t} + U\frac{\partial \Theta}{\partial r} = -\frac{12\kappa}{H^2}\Theta \qquad (6)$$

where κ is thermal diffusivity. We substitute (4) into (5) and (6); after nondimensionalizing H by H_o (a reference thickness), t by $\frac{H_o^2}{12\kappa}$ and r by $\sqrt{\frac{\Delta\rho g H_o^5}{12C\kappa\eta_c}}$, we obtain

$$\frac{\partial H}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r(1+\nu\Theta)H^3\frac{\partial H}{\partial r}\right] \qquad (7)$$

$$\frac{\partial \Theta}{\partial t} = (1+\nu\Theta)H^2\frac{\partial H}{\partial r}\frac{\partial \Theta}{\partial r} - \frac{\Theta}{H^2} \qquad (8)$$

where $\nu = \eta'/\eta_h$ is the only free parameter [see also *Bercovici*, 1992]. These equations are solved numerically via basic finite differences with appropriate Courant conditions on the time step.

Numerical Experiments

In this section we examine gravity currents with 1) constant volume and 2) continuous mass supply from a central conduit. The first case is associated with spreading isolated diapirs, the second to continuously fed lava flows and plume heads. In both cases we consider two different viscosity contrasts, $\nu = 0$ (constant viscosity) and $\nu = 1000$ (representative of silicate materials).

Constant Volume Currents

For constant volume gravity currents, we apply the boundary condition that $\frac{\partial H}{\partial r} = \frac{\partial \Theta}{\partial r} = 0$ at $r = 0$; the boundary condition at the outer radius is arbitrary since we do not allow the gravity current to spread that far. Our initial condition for H is

$$H(r,0) = \begin{cases} \left(1 - r^2/r_N^2\right)^{1/3} & \text{for } r \le r_N \\ 0 & \text{for } r > r_N \end{cases} \qquad (9)$$

where r_N is the initial radius of the current's edge; this condition has the shape of the similarity solution [*Huppert*, 1982] for an isoviscous current. In this case, the thickness scale H_o is simply the starting thickness of the gravity current. The initial condition for Θ uses a super-Gaussian shape

$$\Theta(r,0) = e^{-(r/r_N)^{20}} \qquad (10)$$

which permits nearly isothermal initial conditions in the current but a smooth temperature transition at its edge. As predicted by similarity theory, the isoviscous ($\nu = 0$) current maintains the shape prescribed by (9) as it spreads (Figure 1). In contrast, the variable-viscosity ($\nu = 1000$) gravity current does not conserve shape, becoming very flat and steep-sided (bottom frame, Figure 1). The steepening of the flow front allows the low-viscosity material in the center of the gravity current to push out the cold viscous material near the edge of the current. Since most of this current has low viscosity, it collapses very rapidly, spreading approximately 100 times faster than in the $\nu = 0$ case. Because of the rapid spreading, the nearly isothermal core is retained and stretched outward with the current.

Continuously Fed Currents

The thickness of a gravity current fed by a constant volume flux at its center is singular at $r = 0$. We therefore prescribe an inner vertical boundary at $r = r_i > 0$ (which represents the outer boundary of the supply conduit). The dimensional volume flux through this boundary is a constant Q, and thus

$$r(1+\nu\Theta)H^3\frac{\partial H}{\partial r} = -1 \quad \text{at} \quad r = r_i \qquad (11)$$

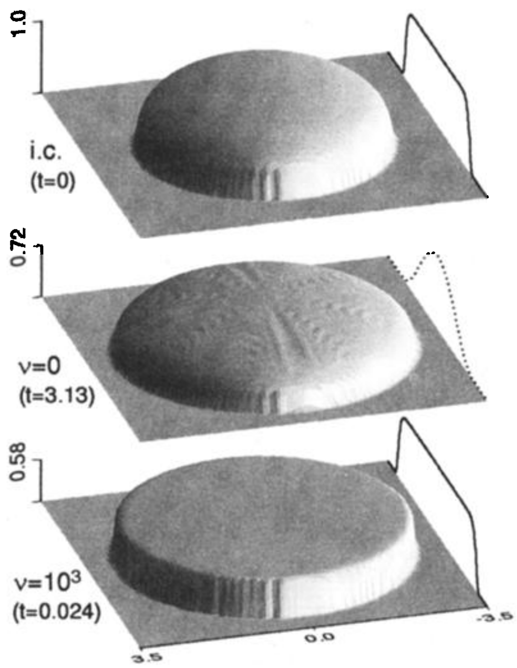

Figure 1. Perspective views of dimensionless thickness H for constant volume gravity currents with different viscosity contrasts ν. Profiles of temperature Θ across the center of the current are shown on the right edge of each frame; the top profile has a maximum value of 1 and subsequent profiles use the same vertical scale. Temperature for the $\nu = 0$ case is scaled up by a factor of 100 and hence shown as a dashed curve. The top frame displays the initial conditions (from (9) and (10) with $r_N = 2.5$). Dimensionless time t, maximum H (rear left corner scale) and ν are indicated.

where we have defined the reference thickness as $H_o = \left(\frac{C\eta_c Q}{2\pi\Delta\rho g}\right)^{1/4}$. We also require that $\Theta = 1$ at $r = r_i$, i.e., we assume fluid is injected at the maximum temperature. The initial condition on H is

$$H(r,0) = \begin{cases} \left(\frac{4}{1+\nu} log(r_N/r)\right)^{1/4} & \text{for } r_i \leq r \leq r_N \\ 0 & \text{for } r > r_N \end{cases} \tag{12}$$

which corresponds to a gravity current at temperature $\Theta = 1$ flowing with constant volume flux (i.e., with (11) applied to the whole fluid). The initial condition on Θ is prescribed by (10). We consider two initial conditions: 1) where there is essentially no pre-existing gravity current (corresponding to direct ejection of fluid against a surface); and 2) where there is a large hot initial current (associated with a starting plume head impinging on a surface while being trailed by a conduit).

The current with no initial volume and $\nu = 0$ essentially maintains the shape prescribed by (12) as it propagates (Figure 2). (Because of the inner vertical boundary, however, this case is not identical to *Huppert's* [1982] similarity solution for the isoviscous gravity current.) During the simulation, the current increases in thickness less than 80% and most of this before $t = 1$. In contrast, the gravity current with $\nu = 1000$ propagates outward as a flat, steep-sided plateau (Figure 2), clearly not conserving the shape prescribed by (12). This current increases in thickness by over 400% and propagates faster than the isoviscous current. The high temperature region of the current, however, does not extend beyond $r \approx 2$ even as the current's edge propagates past this radius. As the edge of the current moves outside the hot low viscosity zone it becomes more rounded.

In the case with a large hot starting volume, the injected volume flux is at first accomodated by the initial spreading of the current (Figure 2). However, most of the current's heat diffuses away fairly rapidly, leaving a small hot core at its center. This has no effect on the $\nu = 0$ current which is therefore not shown. For $\nu = 1000$, a narrow flat plateau begins to inflate within the hot low-viscosity region because a larger hydrostatic head is required to push out the newly cooled high-viscosity fluid. The plateau slowly propagates out toward the edge of the gravity current, and as its front passes into the cold, high-viscosity region it becomes rounder, similar in shape to the isoviscous current. During this time, the plateau quadruples in thickness. Moreover, the edge of the gravity current remains completely static until the plateau front reaches it. Although the choice of initial volume size is somewhat arbitrary, these results are fairly robust. A thicker hot initial gravity current is, at first, too thick to be supported by the volume flux and collapses rapidly to a thinner current (similar to the constant volume current, Figure 1) after which its heat diffuses away and the interior plateau forms as in the above case. If the initial current is thinner the formation of the plateau is simply accelerated.

Small-Slope Caveat

The gravity currents presented here develop steep edges or plateau fronts, especially for large ν. As noted by *Huppert* [1982], gravity current theories are derived

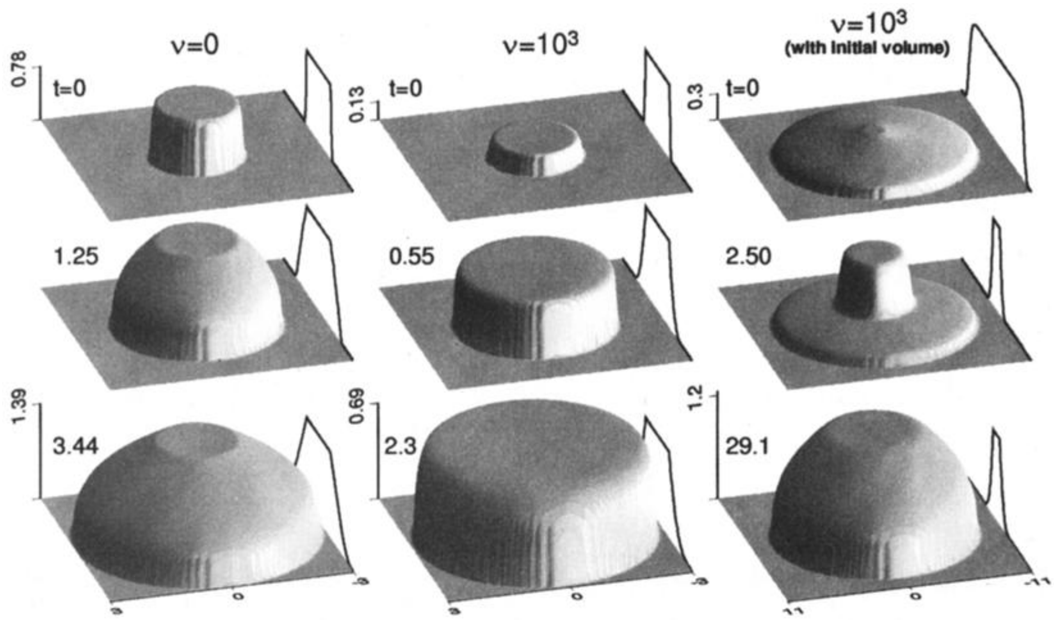

Figure 2. Perspective views of the thickness H (with profiles of Θ on the right edge) of continuously-fed gravity currents for different times t and viscosity contrasts ν. Left and middle columns show currents with essentially no starting volume, i.e. $r_i = 1$ and $r_N = 1.1$. The right column shows a current with a large, hot initial volume, i.e., $r_N = 8.8$. Initial conditions are specified by (12) and (10). Maximum Θ is 1 for all frames. Starting and ending maximum H are indicated by the rear left corner scales.

with the small-slope assumption; i.e., motion within the current is modelled with channel flow. This assumption is far from correct at the edge of the gravity current. Historically, however, these theories have proven suprisingly successful in predicting the steep fronts in laboratory generated gravity currents [e.g., *Huppert*, 1982; *Didden and Maxworthy*, 1982], even for variable rheology [e.g., *Liu and Mei*, 1989]. Thus the channel-flow model appears to capture enough of the important physics to at least partially describe the flow front.

Discussion and Applications

In this paper we have briefly examined a theoretical model for cooling gravity currents with variable viscosity. The model suggests that for a constant volume gravity current, an increase in viscosity contrast ν enhances steepening of the current's edge and flattening of its center. Continuously fed gravity currents with large ν also spread as flat plateaux, or form interior plateaux which propagate to the currents' edge. Furthermore, these gravity currents grow in volume primarily by thickening, as opposed to isoviscous currents which grow almost entirely by spreading [*Huppert*, 1982].

The theory presented here offers a simple model for applications to flow of silicates in geolical settings. The flat plateau shape of variable-viscosity currents is suggestive of lava deposits which lack a dome shape, e.g., mesa lavas [*Cas and Wright*, 1987], *tortas* [*S. Self*, pers. comm., 1994] and possibly even lava domes on Venus [*Pavri et al.*, 1992; *McKenzie et al.*, 1992]. The growth and propagation of interior plateaux within continuously fed gravity currents is indicative of overflow behavior in lava flows, though in that instance the plateaux would essentially undergo wave-breaking which cannot be predicted by this theory. It should be noted that truly accurate models of lava flows probably require incorporation of freezing crust [e.g, *Crisp and Baloga*, 1990].

Gravity current theory also has applications to mantle plume heads spreading beneath oceanic lithosphere [e.g., *Olson*, 1990]. Plumes in the Earth, however, are primarily thermally buoyant, whereas the theory here is more applicable to plume heads with a predominantly chemical density heterogeneity. Moreover, plate motions on Earth tend to draw plume heads into elliptical structures [*Olson*, 1990]. Nevertheless, *Wessel's* [1993] study of the Hawaiian swell suggests a signficantly flattened plateau-like shape for a cross-section of the swell perpendicular to the island chain (after volcanic construction and associated flexure are accounted for); this is reminiscent of the gravity current shapes implied in this paper. Coronae on Venus, which possibly result from mantle plumes, also maintain plateau shapes [*Squyres et al.*, 1992] similar to those suggested by our gravity current theory [see also *Bercovici and Lin*, 1993]. Finally, since the theory of this paper suggests that gravity currents with temperature-dependent viscosity grow by thickening, then it also implies that the available plume-head uplift is significantly greater than that of a constant-viscosity plume head and may provide enough buoyant stress to generate features such as swells and coronae.

Acknowledgments. The author is grateful to Stuart Weinstein, Stephen Self, Peter Olson and John A. Whitehead for useful discussions and/or reviews. This work is supported by NSF grant EAR-9303402.

References

Bercovici, D., Wave dynamics in mantle plume heads and hotspot swells, *Geophys. Res. Lett., 19*, 1791–1794, 1992.

Bercovici, D. and J. Lin, Mantle plume-head waves: A possible mechanism for the formation of the outer-ridges of Venus coronae, *Eos Trans. AGU, 74*, 189, 1993.

Cas, R.A.F. and J.V. Wright, *Volcanic Successions: Modern and Ancient*, Allen & Unwin Publishers, London, Ch. 4, 1987.

Crisp, J. and S. Baloga, A model for lava flow with two thermal components, *J. Geophys. Res., 95*, 1255–1270, 1990

Didden, N. and T. Maxworthy, The viscous spreading of plane and axisymmetric gravity currents, *J. Fluid Mech., 121*, 27–42, 1982.

Fink, J.H. and R.W. Griffiths, A laboratory analog study of the surface morphology of lava flows extruded from point and line sources, *J. Volcanol. Geotherm. Res, 54*, 19–32, 1992.

Huppert, H.E., The propagation of two-dimensional and axisymmetric viscous gravity currents over a rigid horizontal surface, *J. Fluid. Mech., 121*, 43–58, 1982

Liu, K.F. and C.C. Mei, Slow spreading of a sheet of Bingham fluid on an inclined plane, *J. Fluid Mech., 207*, 505–529, 1989.

McKenzie, D., P.G. Ford, F. Liu, and G.H. Pettengill, Pancakelike domes on Venus, *J. Geophys. Res., 97*, 15,967–15,976, 1992.

Olson, P., Hot spots, swells and mantle plumes, in *Magma Transport and Storage*, M.P. Ryan, ed., John Wiley & Sons, 33–51, 1990.

Pavri, B., J.W. Head, K.B. Klose, and L. Wilson, Steep-sided domes on Venus: Characteristics, geologic setting, and eruption conditions from Magellan data, *J. Geophys. Res., 97*, 13,445–13,478, 1992.

Squyres, S.W., D.M. Janes, G. Baer, D.L. Bindschadler, G. Schubert, V.L. Scharpton, and E.R. Stofan, The morphology and evolution of coronae on Venus, *J. Geophys. Res., 97*, 13,611–13,634, 1992.

Wessel, P., Observational constraints on models of the Hawaiian hot spot swell, *J. Geophys. Res., 98*, 16,095–16,104, 1993.

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(received February 16, 1994; accepted March 25, 1994.)